Holographic measurement and synthesis of optical field using a spatial light modulator

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Overview of digital holography

Synthesis with spatial light modulator

Real optical space where optical fields propagate

Measurement by phase-shift interferometry
Motivation of adaptive holographic technology

- Designing hologram
- Synthesizing technology
- Real optical space
- Measuring technology

Adaptive feedback
I. Synthesis of optical field without adaptive feedback
II. Synthesis of optical field with adaptive feedback
III. Synthesis of micro optical field by demagnification optics

Digital holographic technologies

Holographic synthesis

- Synthesis of optical field without adaptive feedback
- Synthesis of optical field with adaptive feedback
- Synthesis of micro optical field by demagnification optics

Holographic measurement

- Measurement of optical field based on phase-shifting interferometry
- Measurement of optical field based on holographic microscopy
I. Spatial light modulator with twisted nematic liquid crystal

1.1 Optimization of twisted nematic liquid crystal SLM

1.2 Wide viewing angle dynamic holographic stereogram
I. Spatial light modulator with twisted nematic liquid crystal

1.1 Optimization of twisted nematic liquid crystal SLM

Ideal amplitude only modulation

Ideal phase only modulation

Typical TNLC modulation with two polarizers.
I. Spatial light modulator with twisted nematic liquid crystal

1.1 Optimization of twisted nematic liquid crystal SLM

Configuration of SLM with TNLC

\[ \theta_{p1} : \text{rotation angle of input polarizer} \]
\[ \theta_{p2} : \text{rotation angle of output polarizer} \]
\[ \theta_{w1} : \text{rotation angle of input wave plate} \]
\[ \theta_{w2} : \text{rotation angle of output wave plate} \]

I. Spatial light modulator with twisted nematic liquid crystal

1.1 Optimization of twisted nematic liquid crystal SLM

Eigenstates of twisted nematic liquid crystal

\[ E_\lambda (+) = \begin{pmatrix} \alpha/(2\gamma^2 + 2\beta\gamma)^{1/2} \\ i(\beta+\gamma)/(2\gamma^2 + 2\beta\gamma)^{1/2} \end{pmatrix} \quad E_\lambda (-) = \begin{pmatrix} (\beta+\gamma)/(2\gamma^2 + 2\beta\gamma)^{1/2} \\ -i\alpha/(2\gamma^2 + 2\beta\gamma)^{1/2} \end{pmatrix} \]

Lu and Saleh model

Coy model
I. Spatial light modulator with twisted nematic liquid crystal

1.1 Optimization of twisted nematic liquid crystal SLM

Optimization of SLM with genetic optimization algorithm

Specifications of system
SLM: SONY LCX016AL-6 (32um pixel pitch)
CCD: KODAK MegaPlus ES1.0/MV (9um pixel pitch)
Laser: Coherent Verdi V-5 (Nd:YAG 532nm)
I. Spatial light modulator with twisted nematic liquid crystal

1.1 Optimization of twisted nematic liquid crystal SLM

Schematics of optimization system

Precision of measuring the phase modulation: $2\pi/50$
I. Spatial light modulator with twisted nematic liquid crystal

1.1 Optimization of twisted nematic liquid crystal SLM

Genetic optimization algorithm

- Genetic algorithm is programmed by Labview™ and Matlab®.
- Main part of genetic algorithm is programmed in Matlab® program.
- Four motorized rotation stages are controlled by Labview™.
- The phase modulation is calculated with interference pattern by a CCD and the amplitude modulation is measured by an OPD.
- These data is delivered into Matlab® with the help of Labview™.
I. Spatial light modulator with twisted nematic liquid crystal

1.1 Optimization of twisted nematic liquid crystal SLM

Phase and amplitude modulations during evolutions

Experimental results before and after genetic optimization

Before the genetic optimization

After the genetic optimization
I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Angular spectrum of object wave and their recording positions

The concave-shaped CGH designed by Benton (1966)
I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Local viewing angles in field of view displayed by a single SLM

Constraint relation in the area and local viewing angles

\[ W_u W_v \theta_u \theta_v / \lambda^2 \approx N \times M. \]
I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Total viewing angles in field of view displayed by a single SLM

\[ \Theta_u \approx 2 \tan^{-1} \left[ \frac{Np}{2f} - \frac{\lambda}{2p} \left( 1 - \frac{d}{f} \right) \right] \]

\[ \frac{\lambda f^2}{Np^2 + (f-d)\lambda} \]

\[ \frac{\lambda f^2}{Np^2 - (f-d)\lambda} \]

Frontward horizontal summit

Backward horizontal summit

Frontward vertical summit

Backward vertical summit

Total viewing angle

Viewing volume

SLM plane

Transfer lens

Focal plane

\[ \lambda, f, d, u, v, \xi, \eta, \Theta, \Theta_u, \Theta_v \]
I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Field of view defined by a curved array of SLMs

\[ \Theta_u' = \frac{(n-1)N'p}{f} + 2 \tan^{-1} \left[ \frac{N'p}{2f} - \frac{\lambda}{2p} \left(1 - \frac{d}{f} \right) \right]. \]

I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Equivalent fields of view

Field of view by three sub-SLMs positioned at different vertical heights in contact with each other diagonally

Field of view by effectively reformed SLM
I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Embodiment of effectively reformed SLM

Structure of effectively reformed SLM

Alignment of some reformed SLMs
I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Schematics of the proposed wide viewing angle dynamic holographic stereogram
I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Pictures of the dynamic holographic stereogram

Curved array of SLMs mounted without upper arms
Whole system with electronic controllers
I. Spatial light modulator with twisted nematic liquid crystal

1.2 Wide viewing angle dynamic holographic stereogram

Computer generated holograms and respective numerical reconstructions

Pictures of the implemented dynamic holographic stereogram

This result is honored to be chosen as ‘Image of the week’, August 4th in OpticsInfoBase.
II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

2.2 Optical implementation of iterative fractional Fourier transform algorithm

2.3 Adaptive beam-shaping with a genetic feedback tuning loop
II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

Optical configuration for phase-shifting interferometry in Fourier optics
II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

The relation between two phases of interferograms in the complex plane

Object and reference waves

\[ U_O = A_o \exp(j\phi) \]
\[ U_R = A_r \exp(j\alpha_i) \]

ith step interferogram

\[ I_i = A_O^2 + A_R^2 + 2A_O A_R \hat{\phi} \cdot \hat{\alpha}_i \]

Conventional 4-step phase-shifting interferometry

\[ U'_O = (I_1 - I_3)/A_R + j^*(I_4 - I_2)/A_R \]

Base of phase-shifting interferometry

\[ I_{ij} \equiv (I_i - I_j)/A_R \]
\[ = 2A_O \hat{\phi} \cdot (\hat{\alpha}_i - \hat{\alpha}_j) \]

General phase-shifting interferometry equation

\[ U'_O = \sum_{i \neq 1} a_i I_{i1} \]

If the difference between two arguments decreases, the magnitude of the base will decrease. This relationship affects the coefficients of bases in phase-shifting interferometry equation.

II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

Original object and twin image in hologram with Fourier optics

The degree of twin image noise can be evaluated by the evenness of reconstructed image on focal plane.

Reconstructed images using only one base of phase-shifting interferometry
II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

The flow of the micro-genetic algorithm to eliminate a twin image

**Genetic algorithm**

1. Create an initial population $x_i = \{\text{Re}\{a_{21}\}, \text{Im}\{a_{21}\}, \text{Re}\{a_{31}\}, \text{Im}\{a_{31}\}, \ldots, \text{Re}\{a_{N1}\}, \text{Im}\{a_{N1}\}\}$

2. Evaluate the fitness function
   \[ f(x_i) = \frac{1}{2} \sum_{m=1}^{N} \left( U_{m}^{\text{cost}} - U_{n}^{\text{max}} \right) \]

3. Determine the elite

4. Mutation

5. Determine new elite

6. Sort the population

7. Exclude the poorest

8. Crossover

9. Update the population

Where $U_{m}^{\text{cost}}$ are modified real Zernike polynomials

**Chromosome**

Coefficients of bases in phase-shifting interferometry equation

Degree of twin image noise

cost function $= \frac{-\sum_{m=0}^{2N} \sum_{n=1}^{2N} U_{m}^{\text{cost}}}{\sum_{m=1}^{2N} \sum_{n=1}^{2N} U_{m}^{\text{max}}}$

Where $U_{m}^{\text{cost}}$ are modified real Zernike polynomials.
II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

Experimental results with the genetic algorithm

Before the genetic algorithm

After the genetic algorithm
II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

Evolutions in the genetic algorithm

![Generated Image]

Fittest chromosome

Resultant evenness and oddness in reconstructed images
II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

Resultant relationship among phase-shift bases

Optimized coefficients of the bases in phase-shifting interferometry.

Resultant phase-shifts of interferograms

As the difference between two arguments decreases, the magnitude of the base also decreases. This relationship affects the coefficients of bases in phase-shifting interferometry equation.
II. Adaptive holographic synthesis with phase-shifting interferometry

2.1 Phase-shifting interferometry with unknown phase-shifts

Comparison of the proposed method with simple filtering method

<table>
<thead>
<tr>
<th>Conventional method</th>
<th>Simple filtering</th>
<th>The proposed method</th>
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</thead>
<tbody>
<tr>
<td>Focused plane</td>
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<td><img src="image6" alt="Defocused plane" /></td>
</tr>
</tbody>
</table>
II. Adaptive holographic synthesis with phase-shifting interferometry

2.2 Optical implementation of iterative fractional Fourier transform algorithm

iterative fractional Fourier transform algorithm

\[ W_n = A(\Phi_n) \exp(j\Phi_n) \]

Hard constraint

Functional relationship between phase and amplitude modulations

Soft constraint

Amplitude and phase freedoms

\[ \text{FRFT} = \text{fractional Fourier transform} \]
II. Adaptive holographic synthesis with phase-shifting interferometry

2.2 Optical implementation of iterative fractional Fourier transform algorithm

Optical implementation of the $a^{th}$ order and its complementary $(2-a)^{th}$ order two-dimensional FRFT

Diverging spherical wave

\[ \exp \left( j\pi \frac{(x_1^2 + y_1^2)}{\lambda R} \right) \]

\[ G(x_1, y_1) \]

\[ P(x_2, y_2) \]

\[ F(x_3, y_3) \]

\[ \frac{\partial}{\partial x} \] for $a^{th}$ order FRFT

\[ \frac{\partial}{\partial y} \] for $(2-a)^{th}$ order FRFT

FL: Fourier lens with a focal length $f$

Associative and the communicative properties

\[ F_{a1} \left[ F_{a2} \left( f \right) \right] = F_{a2} \left[ F_{a1} \left( f \right) \right] = F_{a1+a2} \left( f \right) \]
II. Adaptive holographic synthesis with phase-shifting interferometry

2.2 Optical implementation of iterative fractional Fourier transform algorithm

Optical implementation of the iterative FRFT algorithm

II. Adaptive holographic synthesis with phase-shifting interferometry

2.2 Optical implementation of iterative fractional Fourier transform algorithm

Schematics of the optical iterative FRFT system

BS: Beam splitter
M: Mirror
P: Polarizer
BE: Beam expander
FL: Fourier lens with a focal length f
PL: Positive lens
NL: Negative lens
TNLC: Twisted nematic liquid crystal
S: Mechanical shutter
Att.: Optical attenuator
PM: Patterned mask

Measurement part: phase shifting digital holography

4/f system for matching scaling factors

1st beam path
2nd beam path

Drive the piezo stage
Detect the phase profiles
Encode the phase profiles
Control the shutters

Laser
S
SBSBE
ff
PM P

Piezo stage

CCD

4-f system

Center for Active Plasmonics Application Systems
II. Adaptive holographic synthesis with phase-shifting interferometry

2.2 Optical implementation of iterative feedback loop

Multiplexing four DOE phase profiles and their diffraction images

Patterned masks

Applied spherical phase profiles

Diffraction images of the multiplexed DOE captured by a CCD
II. Adaptive holographic synthesis with phase-shifting interferometry

2.3 Adaptive beam-shaping with a genetic feedback tuning loop

Schematics of the system

II. Adaptive holographic synthesis with phase-shifting interferometry

2.3 Adaptive beam-shaping with a genetic feedback tuning loop

Program panel

- Phase profile in hologram
- Zernike polynomials
- Phase modulation curve
- Coefficients of Zernike polynomial
- Difference image between A and B
- Cost value of the elite
- A. Diffraction image
- B. Target image
II. Adaptive holographic synthesis with phase-shifting interferometry

2.3 Adaptive beam-shaping with a genetic feedback tuning loop

Optimization of encoding index in SLM

SLM: SONY LCX016AL-6 (32um pixel pitch)
II. Adaptive holographic synthesis with phase-shifting interferometry

2.3 Adaptive beam-shaping with a genetic feedback tuning loop

Experimental results for optimized encoding index

![Graph showing encoding index vs. designed phase modulation](image)

- 1st
- 43rd
- 299th

Diffraction image by the SLM with the linear encoding index

Diffraction image by the SLM with the optimized encoding index
II. Adaptive holographic synthesis with phase-shifting interferometry

2.3 Adaptive beam-shaping with a genetic feedback tuning loop

Experiments for compensation of aberration
II. Adaptive holographic synthesis with phase-shifting interferometry

2.3 Adaptive beam-shaping with a genetic feedback tuning loop

Experimental results for compensation of aberration

Target diffraction image  Evolution of the diffraction image  Resultant Zernike phase profile
II. Adaptive holographic synthesis with phase-shifting interferometry

2.3 Adaptive beam-shaping with a genetic feedback tuning loop

Experimental results for compensation of aberration

Diffraction image with aberration

Compensated diffraction image
III. Holographic synthesis and measurement in micrometer scale

3.1 Micro field synthesis

3.2 Holographic angular spectrum analyzer
III. Holographic synthesis and measurement in micrometer scale

3.1 Micro field synthesis by demagnification

Demagnification of optical field without speckle noise

Fourier optics

SLM

FT lens

Optical field

\[ p : \text{pixel pitch} \]
\[ N : \text{pixel number} \]
\[ W_N : \text{Nyquist width} \]
\[ \delta : \text{resolution} \]

\[ f \]

\[ f \]

4f optics

SLM

4f system

Optical field

\[ p : \text{pixel pitch} \]
\[ N : \text{pixel number} \]

Blurred image with the same resolution

\[ f \]

\[ f \]

\[ f \]

\[ f \]
III. Holographic synthesis and measurement in micrometer scale

3.1 Micro field synthesis by demagnification

Demagnification of optical field by SLM in with telecentric lens

Specifications of system
Telecentric lens: Edmund gold series 0.09x
SLM: Epson device L3P06X (12um pixel pitch)
CCD: KODAK MegaPlus ES1.0/MV (9um pixel pitch)
Laser: Coherent Verdi V-5 (Nd:YAG 532nm)

J. Hahn, Y. Lim, H. Kim, and B. Lee, J. of Holography and Speckle (accepted for publication) (Invited paper).
III. Holographic synthesis and measurement in micrometer scale

3.1 Micro field synthesis by demagnification

Holographic measurement for micro optical field

Specifications of system
Telecentric lens: Edmund gold series 0.09x
SLM: Epson device L3P06X (12um pixel pitch)
CCD: KODAK MegaPlus ES1.0/MV (9um pixel pitch)
Laser: Coherent Verdi V-5 (Nd:YAG 532nm)
Objective: LMPlanFLN-BD 10x N.A. 0.25
PZT: Piezo system Jena XYZ-38
III. Holographic synthesis and measurement in micrometer scale

3.1 Micro field synthesis by demagnification

Holographic measurement of micro optical field with SLM
III. Holographic synthesis and measurement in micrometer scale

3.1 Micro field synthesis by demagnification

Micro optical field with SLM in the amplitude modulation condition
III. Holographic synthesis and measurement in micrometer scale

3.1 Micro field synthesis by demagnification

Micro optical field with SLM in the phase modulation condition

Spherical lens with focal length 6.78mm

Phase profile of signal wave

Numerically reconstructed wave at 6.89mm
III. Holographic synthesis and measurement in micrometer scale

3.2 Holographic angular spectrum analyzer

Holographic microscopy for analyzing angular spectrum
III. Holographic synthesis and measurement in micrometer scale

3.2 Holographic angular spectrum analyzer

Holographic microscopy for analyzing angular spectrum

Specifications of system
CCD: SONY XCD-SX90 (3.75um pixel pitch)
Laser: Coherent Verdi V-5 (Nd:YAG 532nm)
Pinhole: Newport 5um aperture
Objective: MPlanFLN-BDP 100x N.A. 0.9
PZT: Piezo system Jena XYZ-38
III. Holographic synthesis and measurement in micrometer scale

3.2 Holographic angular spectrum analyzer

Transmission characteristics of objectives

<table>
<thead>
<tr>
<th>Products</th>
<th>Magnification</th>
<th>N.A.</th>
<th>W.D. (mm)</th>
<th>Resolution (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPLAN FLN-BDP</td>
<td>100x</td>
<td>0.9</td>
<td>1.0</td>
<td>0.37</td>
</tr>
<tr>
<td>LMPLAN FLN-BD</td>
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<td>0.8</td>
<td>3.3</td>
<td>0.42</td>
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<tr>
<td>LMPLAN FLN-BD</td>
<td>50x</td>
<td>0.5</td>
<td>10.6</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Transmission profile of objective (MPLAN FLN-BDP 100x) for compensation
III. Holographic synthesis and measurement in micrometer scale

3.2 Holographic angular spectrum analyzer

Holographic measurement of angular spectrum

SEM images of pyramidal shape structure
Bottom side = 47.5 µm  
Height = 3.4 µm
III. Holographic synthesis and measurement in micrometer scale

3.2 Holographic angular spectrum analyzer

Fresnel domain filter at stop plane

At this position, Fresnel domain filter is located.
III. Holographic synthesis and measurement in micrometer scale

3.2 Holographic angular spectrum analyzer

Perspective views with local angular spectrums

Original view

Top view

Right view

Left view

Bottom view
Technical relation of my researches in digital holography

**Holographic synthesis**
- Optimization of SLM
- Holographic stereogram
- Micro field synthesis

**Holographic measurement**
- Phase-shifting interferometry
- Holographic microscopy
- Angular spectrum analyzer

**Adaptive optical systems**
- Optical IFRFTA
- Adaptive beam shaping
Phase-shifting interferometry with three CCDs

The effects of longitudinal and transversal displacement in interferograms

Reconstructed image from the experiments under longitudinal vibrations

Reconstructed image from the experiments under transversal vibrations