Digital refraction distortion correction with an astigmatic coherence sensor

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We demonstrate the sensing and correction of an isoplanatic refractive distortion (not lens aberrations), using the complete measurement of the partially coherent field in an aperture that the previously described astigmatic coherence sensor provides. Isoplanatic distortions, and in general distortions that do not cause energy loss, maintain the orthogonality of the coherent modes. We use the fact that a common distortion will occur to all coherent modes to separate the distortion from the source behind it, rather than requiring a reference source at a different wavelength. Digital deconvolution was performed on the full four-dimensional partially coherent field for simultaneously computing the distortion and the source intensity distribution. © 2002 Optical Society of America

1. Introduction
Distorted wave fronts have been characterized with interferometry, holography, phase diversity, modal analysis, and wave-front sensors. A distorted wave front can be corrected with adaptive optics, liquid-crystal phase modulators, or digital deconvolution from wave-front sensing. We describe here a method of digital wave-front sensing and deconvolution based on the recently described astigmatic coherence sensor (ACS). The ACS measures the four-dimensional (4-D) spatial coherence function within an aperture. In this paper we use the ACS for digitally characterizing and correcting a pupil wave-front distortion (not those caused by third-order aberrations) in an imaging system. This method is unique in that it employs an extremely powerful tool for analyzing partially coherent sources, the coherent-mode decomposition. This method can be employed only when all the correlation relations are measured between a set of points, as the ACS does.

The ACS consists of a lens combination that has an adjustable horizontal-to-vertical focal-length ratio. A sensor array is placed behind the lenses and measures the intensity of the field. The intensity $I(x, y, z)$ is measured in the $x$–$y$ plane transverse to the optical axis and as a function of $z$, the distance from the sensor array to the principal plane of the lens combination. $I(x, y, z)$ is related to the mutual intensity of the quasi-monochromatic field of wavelength $\lambda$ in the aperture, $J(\xi - \Delta x, \eta - \Delta y, \xi + \Delta x, \eta + \Delta y)$, by a 4-D Fourier transform:

$$z^2 I \left( \frac{x}{z}, \frac{y}{z}, \frac{1}{f_x} - \frac{1}{z}, \frac{1}{f_y} - \frac{1}{z} \right) = \frac{1}{2} \int_A \int_A \frac{J(\Delta x, \Delta y, q_x, q_y)}{\Delta x \Delta y} d\xi d\eta + 4q_x \left( \frac{1}{f_x} - \frac{1}{z} \right) + 4q_y \left( \frac{1}{f_y} - \frac{1}{z} \right) \right] \times \exp \left\{ \frac{2\pi}{\lambda} \left( -4x\Delta x - 4y\Delta y \right) \right\} \times dq_x dq_y d\Delta x d\Delta y, \tag{1}$$

where $f_x$ and $f_y$ are the focal lengths of the astigmatic lens combination along the $x$ and the $y$ axes, $q_x = \hat{x} \Delta x$, and $q_y = \hat{y} \Delta y$. For an incoherent source, the domain of $J(\cdot)$ reduces to three dimensions, because incoherent sources produce only independent sphera-
tical waves. However, when incoherent sources are imaged through a distortion, the distorting medium can break the symmetry of spherical waves and produce independent data in a 4-D domain. In some cases knowledge of the 4-D mutual intensity can be used to recover an undistorted image of the original source. In this paper we use measurements of $J(\cdot)$ in four dimensions to compensate for an isoplanatic (inside the pupil) phase distortion.

2. Power-Preserving Distortions

The coherent-mode decomposition expresses a partially coherent field as an orthogonal expansion of analytic wave fronts from uncorrelated sources: $J(r_1, r_2) = \sum \lambda_i \phi_i(r_1) \psi_i^*(r_2)$, where the eigenvalues $\lambda_i$ correspond to the power emitted from each source and the orthogonal eigenfunctions $\phi_i(r)$ are the complex wave fronts from each source in the entrance pupil $S$. We consider the transmission of a partially coherent wave front through an optical system from an entrance pupil $S$ to an exit pupil $S'$. The optical system has a coherent transfer function $H(r_1, r_1') = \sum S \psi_i(r_1) \phi_i^*(r_1')$, where the $S_i$ are positive singular values and the $\phi_i(r_1)$ and $\psi_i(r_1')$ are the orthogonal singular functions in $S$ and $S'$, normalized to 1. The functions $\phi_i(r_1') = \sum S \psi_i(r_1) \phi_i^*(r_1')$ are the wave fronts in the exit pupil of the coherent wave fronts for each coherent mode. We can then calculate the overlap matrix $P_{ij} = \int S \phi_i(r) \psi_j^*(r) dr$ are the singular values $S_i$ correspond to the fraction of power in the entrance wave front that projects onto the singular function $\psi_i(r)$ that exits the pupil. If no power is lost, the $S_i = 1$ and the $P = I$, the identity matrix, so all the coherent modes are orthogonal upon leaving the exit pupil. However, loss of power in general leads to a loss of orthogonality among the coherent modes. As a result, sources that are uncorrelated appear partially correlated in the aperture, owing to a loss of information. The coherent modes at the exit pupil will no longer correspond one-to-one with input modes. This can be seen when we look at two closely spaced point sources through an aperture too small to resolve them. For a phase distortion placed at the exit pupil, no power can be scattered out of the optical system, and so orthogonality is preserved. More-general distortions such as anisoplanatic distortions will usually scatter light away from the exit pupil, and the coherent modes from which the most light is diverted will probably be “mixed” the most with other modes.

Incoherent point sources that radiate spherical waves at the same distance from the entrance pupil can be assumed to produce different coherent modes (orthogonal wave fronts) when there is negligible overlap between the images of the points at the image plane (excluding power loss, distortion, or aberrations in the intervening optical system). Even if there are aberrations, power is usually not lost if there are no limiting stops in the optical system, so the aberrated wave fronts still have orthogonal wave fronts. As long as one knows that a given optical system is power preserving and that independent sources radiate to different coherent modes, a coherence-mode decomposition will separate the wave fronts due to independent sources even without knowledge of the exact transformation between the source and the exit pupil. By examining the wave fronts that result from several independent sources, one may be able to infer information about the inter-}\n
\begin{equation}
\text{vening optical system. For example, if a phase distortion is placed in the pupil of an optical system viewing a planar incoherent source, all the coherent modes upon exit will have the same planar distortion on them. From the coherent modes one may be able to deduce simultaneously the distortion and the source behind it, even when the sources cannot be imaged separately or turned on--off in sequence. When the pupil is too small, or intervening blockages or distortions absorb or divert power, the coherent modes can be expected to mix in a way that varies continuously with the amount of lost power from the source. One may be able to bound the amount of cross talk between coherent modes given a particular probabilistic model of the distortion. Quantifying this “mixing” of the coherent modes based on the type of distortion is beyond the scope of this paper.}

3. Experimental Verification

Our goal is to image a test object through an unknown distortion. As illustrated in Fig. 1, we use the ACS to measure the mutual intensity that results from the test object and a point reference source propagation through a thin distor-}\n
\begin{equation}
\text{ter. Both sources are quasimonochromatich with the same nominal wavelength. The orthogonality of the spatial modes is used to separate the point source and object, whereas in adaptive optics the reference would be distinguished by its wavelength. In theory, a point source should not be necessary if the test object contained spatial frequencies up to the bandlimit of the system. The coherent modes would then correspond to different points on the object. However, the thickness of}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Diagram of the setup of the source and ACS.}
\end{figure}
the lines of the test object are much larger than the resolution limit of the system and so do not function well as a point source.

The mutual intensity \( J(x_1, y_1, x_2, y_2) \) of the combined source is the sum of the mutual intensity resulting from the point reference and the test object. To separate the contributions of the two, we use the coherent-mode decomposition. \(^{18,19} \) The coherent-mode decomposition expands the mutual intensity as a sum of coherent fields multiplied by uncorrelated random variables: 
\[
J(x_1, y_1, x_2, y_2) = \sum_i \lambda_i \phi_i(x_1, y_1) \phi_i^*(x_2, y_2). \]
Since the fields produced by the point source and the illuminated transparency will be uncorrelated, we can apply the coherent-mode decomposition to the mutual intensity to find the field that results from the point source alone.

We assume that the distortion is isoplanatic with a pupil function \( T(x, y) \) in the aperture. This transformation transforms the mutual intensity into 
\[
J_p(x_1, y_1, x_2, y_2) = J(x_1, y_1, x_2, y_2)T(x_1, y_1)T^*(x_2, y_2). \]
Since the point source would produce a spherical-wave coherent-mode without the distortion, we can use the actual coherent mode of the point source to characterize the distortion \( T(x, y) \). After performing the coherent-mode decomposition on \( J_p(x, y) \), we find the coherent mode \( \phi_1(x, y) \) corresponding to the distorted point source and form the conjugate phase distortion \( \phi_1^*(x, y) \). We then find an estimate of the coherence function before distortion 
\[
J'(x_1, y_1, x_2, y_2) = J_p(x_1, y_1, x_2, y_2)\phi_1(x_1, y_1)\phi_1^*(x_2, y_2). \]
This estimate has the distortion corrected for the entire field including the test transparency and also conjugates the phase that results from propagation of the field. We then use a 4-D inverse Fourier transform to recover the intensity that an imaging system would have measured at the in-focus plane, which we expect will be the undistorted image of the source.

Our implementation of the ACS uses three lenses. Two of the lenses are cylindrical and of 300-mm focal length with their focal axes aligned and rotated to an angle \( \theta/2 \) from the horizontal axis. The third lens is a 150-mm focal-length cylindrical lens and is placed between the other two with its axis placed \( -\theta/2 \) from the horizontal axis. Together they form a lens with horizontal focal length 
\[
1/f_x = 2 \cos^2(\theta/2)/f \quad \text{and} \quad 1/f_y = \sin^2(\theta/2)/f \quad \text{vertical focal length}, \]
where \( f = 150 \) mm. The reference point source in our experiments was a 4-mW 660-nm laser diode attenuated by three neutral-density filters with a total optical density of 4.6. Unlike a normal reference point source as used in adaptive optics, we used spatial coherence rather than wavelength to distinguish the reference and test object. Rather, the point source was incorporated to include an object in the scene of sufficiently high spatial bandwidth to reconstruct the distortion. The test object was a laser-printer transparency made diffuse by means of rubbing the back surface with sandpaper, with transparent letters “UI” surrounded by a black opaque background. This object was rear illuminated by seven red LEDs. The light from the test object and the point reference was combined with a beam splitter. The objects were approximately 30 cm from the aperture of the ACS. The distortion plate was an approximately 5 \( \times \) 5 cm square of 2-mm-thick transparent acrylic, which was softened by heating and twisted into a distorting shape. It was placed \( -15 \) cm from the front of the ACS to make the source not resolvable by a standard stigmatic imager. Figure 1 shows a diagram of the source and the ACS. Figure 2 shows a picture of the source taken through the distortion before correction. We note that with the distortion, the images of the laser diode and transparency are inseparable.

The pixel pitch of the CCD sensor was \( p = 19 \) \( \mu \) m, and it was nominally located \( l = 185 \) mm from the principal plane of the ACS. To acquire intensity data, \( \theta \) and the CCD range were adjusted so that the defocus parameters \( 1/f_x - 1/z \) and \( 1/f_y - 1/z \) were stepped through combinations of 64 positions 0.034 m\(^{-1}\) apart to sample a total range of 2.13 m\(^{-1}\). At each defocus setting, a 64 \( \times \) 64 region of the CCD was sampled. The size of the sampled region was proportional to the distance \( z \) from the principal plane. A band-limiting interpolator resampled the pixels to have a spacing slightly above or below 19 \( \mu \) m as needed. The resolution of the measured coherence was \( d = \lambda/Np = 96 \) \( \mu \) m. Ultimately 4096 images were recorded in 12 h.

We recovered a deblurred image from the coherence data by the following steps:

1. The data from the ACS were measured as a 64 \( \times \) 64 \( \times \) 64 \( \times \) 64 discretized version of \( I(x/z, y/z, 1/f_x - 1/z, 1/f_y - 1/z) \). The sampling rate in the \( x/z \) and \( y/z \) variables was 1.02 \( \times \) 10\(^{-4}\), and the sampling rate in \( 1/f_x - 1/z \) and \( 1/f_y - 1/z \) was 0.034 m\(^{-1}\). With a radix-2 4-D real-to-complex fast Fourier transformation this was converted to the discrete coherence samples \( J(\Delta x, \Delta y, \lambda \Delta x, \lambda \Delta y)/|\Delta x \Delta y| \). This data set was only 32 \( \times \) 64 \( \times \) 64 \( \times \) 64 because it was the Fourier transform of a real function.
2. We then multiplied the 4-D coherence by a filter $|\Delta x \Delta y|$ to eliminate the $\Delta x \Delta y$ factor in Eq. (1). To avoid the loss of this data for $J(\cdot)$ such that $\Delta x = 0$ or $\Delta y = 0$, we multiplied by a small number (0.5) instead of zero.

3. We used the Lanczos sparse matrix eigenvalue algorithm to find the coherent-mode decomposition of $J(\cdot)$. We did this by discretizing the integral that defines the coherent-mode decomposition,

$$\phi(\mathbf{r}_2) = \lambda_k \int_A \phi(\mathbf{r}_1)J(\mathbf{r}_1, \mathbf{r}_2) d\mathbf{r}_1 \rightarrow \phi_j$$

$$= \lambda_k \sum_j \phi_j J(\mathbf{r}_1 - \mathbf{r}_j, \frac{\mathbf{r}_i + \mathbf{r}_j}{2}),$$

(2)

where $\mathbf{r}_i$ are the positions of the points of interest in the aperture, the $\lambda_i$ are the eigenvalues, and $J(\cdot)$ is a function of the center and difference positions of the correlated points. In the Lanczos algorithm, the sampling points of $J(\cdot)$ and $\phi(\cdot)$ do not coincide, because they are measured on separate coordinate systems. To perform the matrix–vector multiply step in the algorithm, a quadrilinear interpolator was used to find samples of $J(\cdot)$ from the 16 nearest samples.

4. We found the $64 \times 64$ sampled field corresponding to the highest eigenvalue coherent mode $\phi_1(x, y)$, which was due to the laser diode. This field is sampled with a period of 96 μm. The real part of this analytic field is shown in Fig. 3, and the image dimensions are $6.1 \times 6.1$ mm.

5. We computed an inverse filter $\phi'(x, y)$ from this, using the following:

$$\phi'(x, y) = \frac{\phi_1^*(x, y)}{|\phi_1(x, y)| + 0.01\phi_{max}}.$$

(3)

This essentially produced a filter with a conjugate phase to the distortion. The $\phi_{max}$ term is the maximum magnitude of the field within the aperture and was added to suppress noisy field points with low magnitudes. This is a noise-reduction approach similar to that of a Wiener filter.

6. We formed a nonblurred sampled coherence function:

$$J'(\Delta x, \Delta y, \hat{x}\Delta x, \hat{y}\Delta y) = J(\Delta x, \Delta y, \hat{x}\Delta x, \hat{y}\Delta y)\phi'(\hat{x} - \Delta x, -\Delta y) \times \phi'^*(\hat{x} + \Delta x, \hat{y} + \Delta y)/|\Delta x \Delta y|. \quad (4)$$

The denominator was added to reverse step 2 so that we can convert the result back to intensity data. When $\Delta x$ or $\Delta y$ is zero, we multiply the sample by 2 to reverse the 0.5 factor of step 2 for these points. Since, as in step 3, the samples of $J(\cdot)$ and $\phi(\cdot)$ do not coincide, a bilinear interpolation of $\phi'(\cdot)$ is performed between the nearest four neighbors of a needed point.

7. We used a 4-D inverse complex-to-real fast Fourier transform to reverse step 1 and recover $I(x/z, y/z, 1/f_x - 1/z, 1/f_y - 1/z)$.

8. Finally, the plane of data corresponding to the in-focus image was extracted and is displayed in Fig. 4.

We believe that the legibility of the letters has been substantially improved. Also note that the reference diode is pointlike, meaning that the blurring of the laser diode has been successfully removed. The image of the diode is not a single pixel, because the bandwidth of the imaging system was too small to image it as one pixel.

We have demonstrated that the 4-D coherence of a nonstigmatic source such as illuminated blurred text can be sampled by the ACS. With the entire coherence sampled, we can digitally simulate the propaga-
tion of partially coherent light to apply an inverse filter and recover the unblurred image. We believe that these methods represent a powerful application of coherence theory to imaging and may ultimately benefit microscopy, astronomical imaging, and imaging through turbulence. Since detailed knowledge of the coherent-transfer function of the system may not be required a priori, measurement of the 4-D coherence and subsequent decomposition may provide a powerful way of passively inferring details about intervening optical systems.

References