ECE 299 Holography and Coherent Imaging

Lecture 3 Three exercises in hologram analysis

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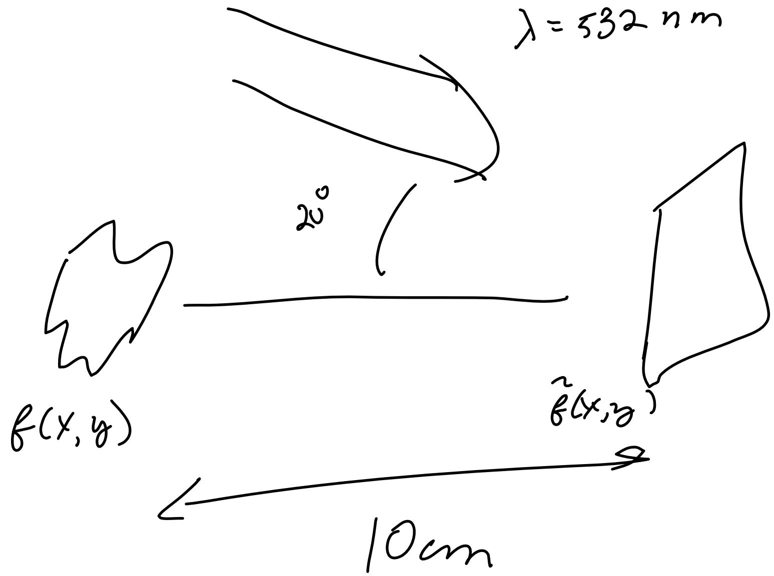
Outline

1. Resolution of off axis holograms
2. Holographic magnification
3. Phase and amplitude holograms

Problem 1. Resolution of off axis holograms

A hologram is recorded of a object at a range of 10 cm using 532 nm light. The angle between the reference and signal beams is 20 degrees.

1. Estimate the maximum fringe frequency on the holographic plate
2. Estimate the angular resolution of the holographic image
3. Estimate the spatial resolution of the image at the object plane
4. How large should the holographic plate be?
5. What sampling period is required to simulate this system? How large an object can be simulated?



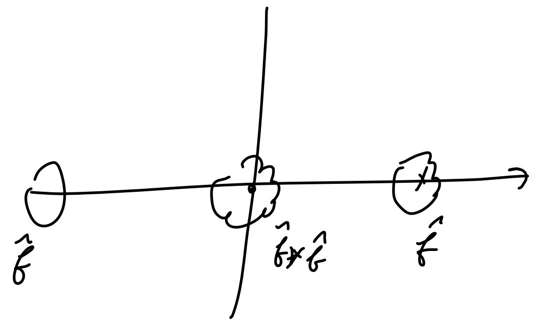
$$I = \left| A e^{i 2\pi \frac{\sin\theta}{\lambda} x} + \tilde{f}(x,y) \right|^2$$

$$= |A|^2 + |\tilde{f}|^2 + A e^{i 2\pi \frac{\sin\theta}{\lambda} x} \tilde{f}^*(x,y)$$

$$+ \text{c. c.}$$

$\hat{I}(u,v)$

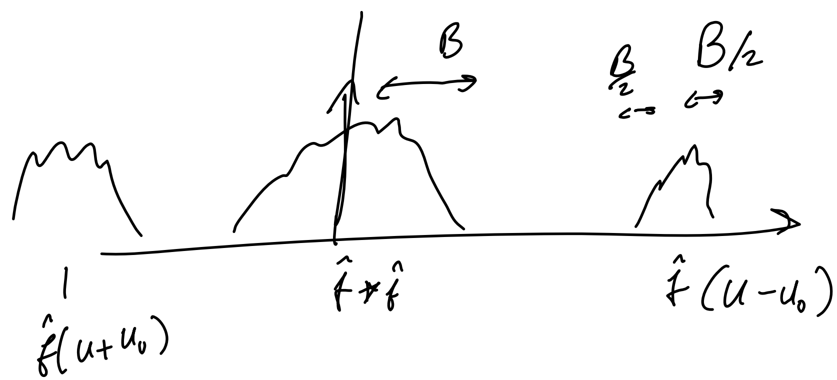
1.



$$u_0 = \frac{\sin\theta}{\lambda} = 6.4\cdot 10^5\ m^{-1}$$

$$= 642\ mm^{-1}$$

$\hat{I}(u,0)$



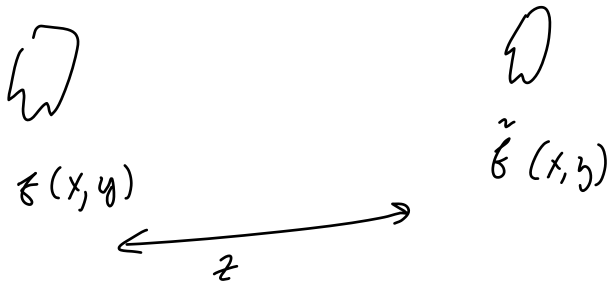
$$U_{max} = \frac{B}{2} + u_0 \qquad \frac{3B}{2} \le u_0$$

$$U_{max} \le \frac{u_0}{3} + u_0 = \frac{4u_0}{3} = 857\ \text{lp/mm} \qquad r = 1.2\ \text{um}$$

Angular resolution

$$B \leq \frac{2}{3} u_0 = 429 \text{ lp/mm}$$

$\delta x \approx$



$$\mathcal{F}\{\tilde{f}(x,y)\} = \hat{f}(u,v)\, e^{i\pi\lambda z(u^2+v^2)}$$

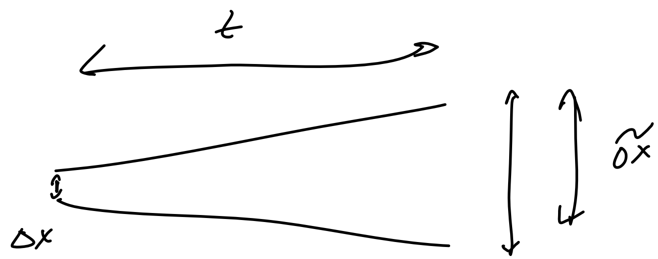
$$\delta x \approx \frac{1}{B} \approx 2.3\ \mu m$$

$$\delta\theta = \frac{\delta x}{z} = 23\ \mu rad \approx 5 \text{ arc seconds}$$

Holographic plate size.

$$I(x) = \left| R e^{i2\pi u x} + \tilde{f}(x) \right|^2 P(x)$$

$$= P(x)\left[|R|^2 + |f|^2 \right] + R e^{i2\pi u x} \tilde{f}(x) P(x) + c.c.$$

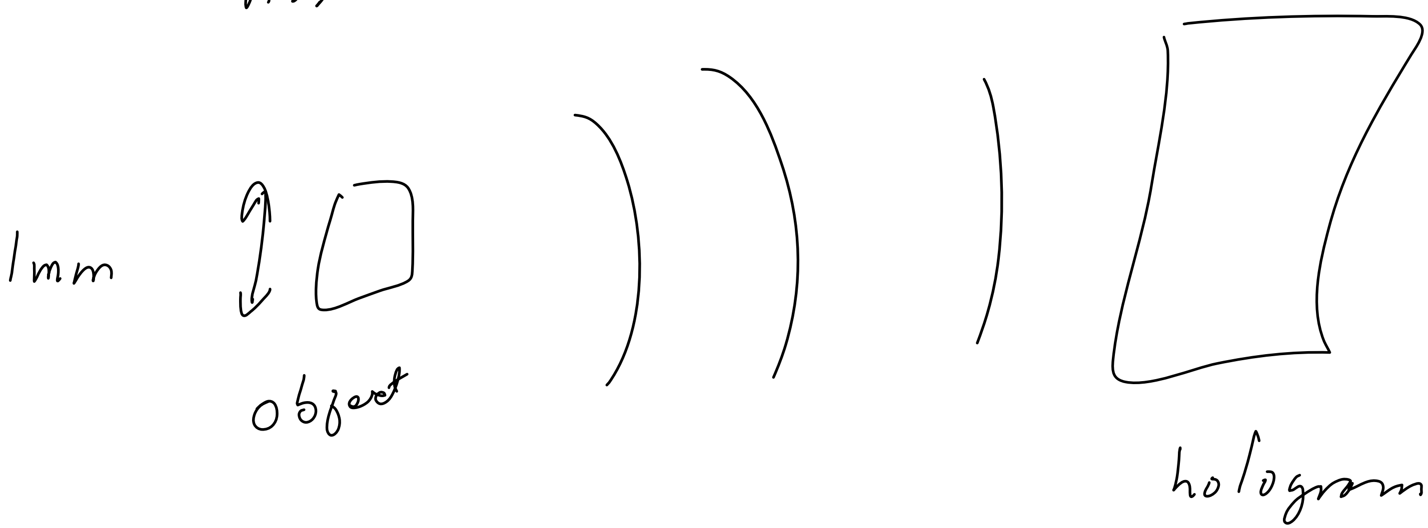


$$\tilde{\Delta x} \approx \Delta x + \frac{\lambda z}{\Delta x} \approx 22 \text{ mm}$$

Sampling period

$\Delta x = 1.1\ \mu m$

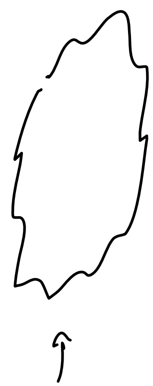
Assume $N = 1024$



Problem 2. Magnification

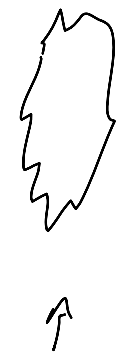
1. Describe holographic magnification using a change of wavelengths
2. By what factor may one reasonably magnify an object at range R using holography?

Holographic magnification



boundary condition

$f(x,y)$

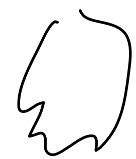


diffracted field

$$\tilde{f}(x',y') = \iint f(x,y)\, e^{i\frac{\pi}{\lambda z}\left((x'-x)^2 + (y'-y)^2\right)}\, dx\, dy$$

Holographic magnification

Hologram

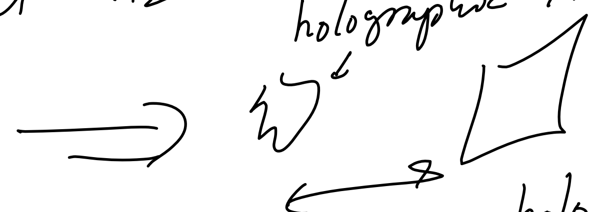


object

$$\check{f} = \mathcal{F}\left\{\hat{f}(u,v)\, e^{i\pi\lambda_1 z_1 (u^2+v^2)}\right\}$$

reconstruction at λ_2

holographic image



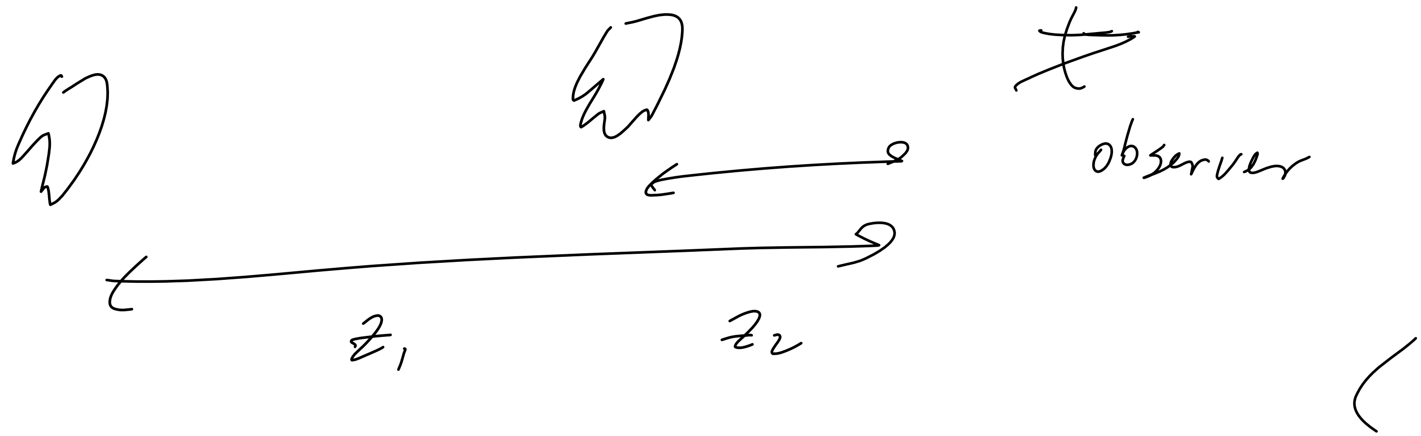
hologram

z_2

Image field

$$\mathcal{F}^{-1}\left\{\hat{f}(u,v)\, e^{i\pi\lambda_1 z_1 (u^2+v^2)}\, e^{-i\pi\lambda_2 z_2 (u^2+v^2)}\right\}$$

$$\lambda_1 z_1 = \lambda_2 z_2$$



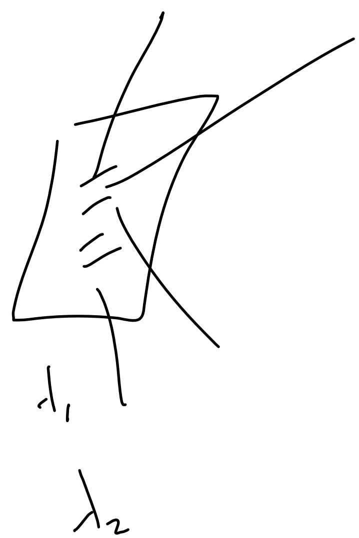
object appears $\frac{\lambda_2}{\lambda_1}$ times closer

and is thus magnified in angular extent

Holographic microscopy

Can one use holography to see subwavelength features?

Version 1 loss of information on propagation

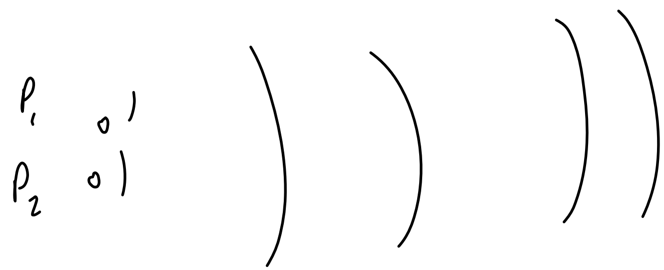


maximum spatial frequency is

$$\frac{1}{\lambda}$$

Holographic Microscopy

Can one use holography to observe sub wavelength features?



$$e^{i2\pi\frac{(x-x_1)^2+y^2}{\lambda z}} + e^{i2\pi\frac{(x-x_2)^2+y^2}{\lambda z}}$$

hologram must resolve frequency

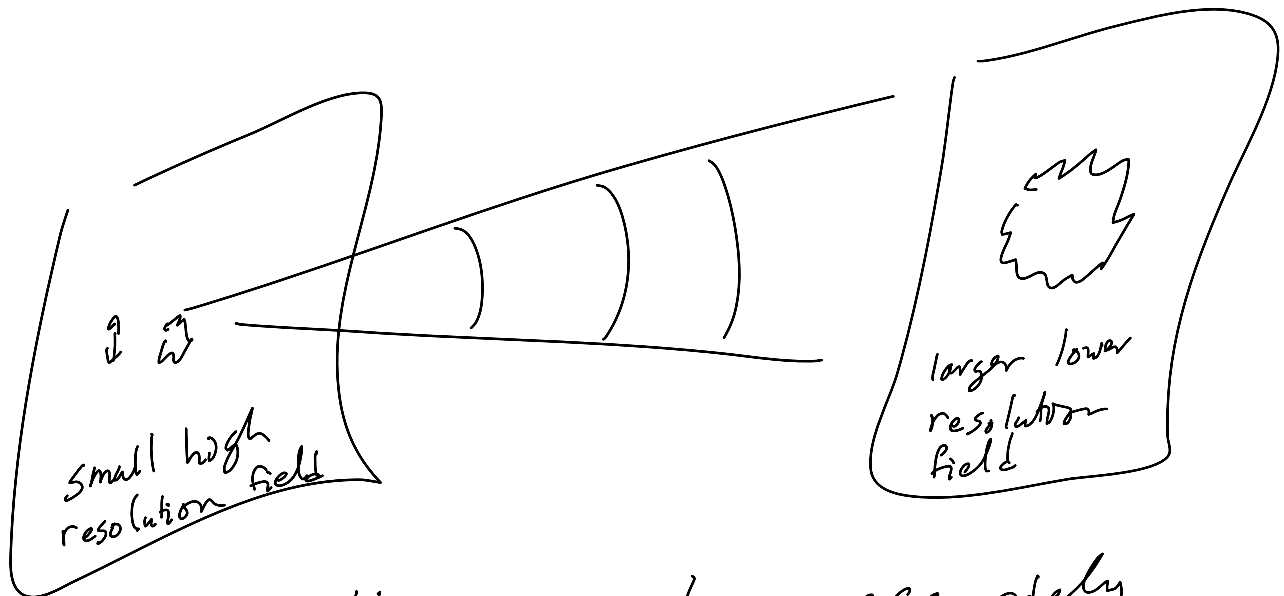
$$u = \frac{\Delta x}{\lambda_1 z}$$

$$\Delta x < \frac{\lambda_1 z}{2\lambda_2}$$

Size of hologram must be $D > \frac{\lambda z}{\Delta x}$

frequency must be $\leq \frac{1}{2\lambda_2}$

Conservation of Space-Bandwidth product



would resampling accurately describe field propagation?

Problem 3. Phase and amplitude holograms

1. What is the maximum diffraction efficiency for an amplitude hologram?
2. What are the relative diffraction efficiencies for higher orders of a phase hologram?

Amplitude hologram

→ commonly made using silver halide emulsions

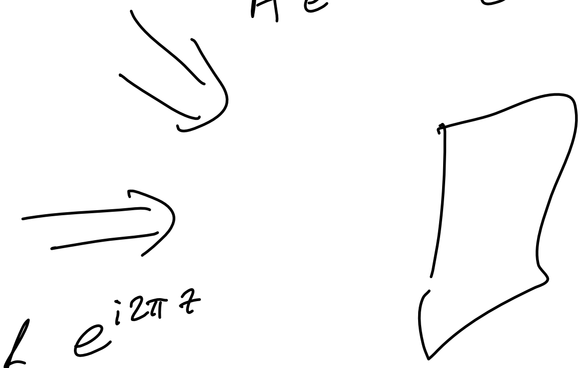
→ transmission $t(x,y)$ is real

$$\emptyset \le t(x,y) \le 1$$

$$t \propto |r|^2 + |f|^2 + f r^* + r f^*$$

Example amplitude hologram

$$A e^{i2\pi u x} e^{i2\pi\sqrt{\frac{1}{\lambda^2}-u^2}\, z}$$



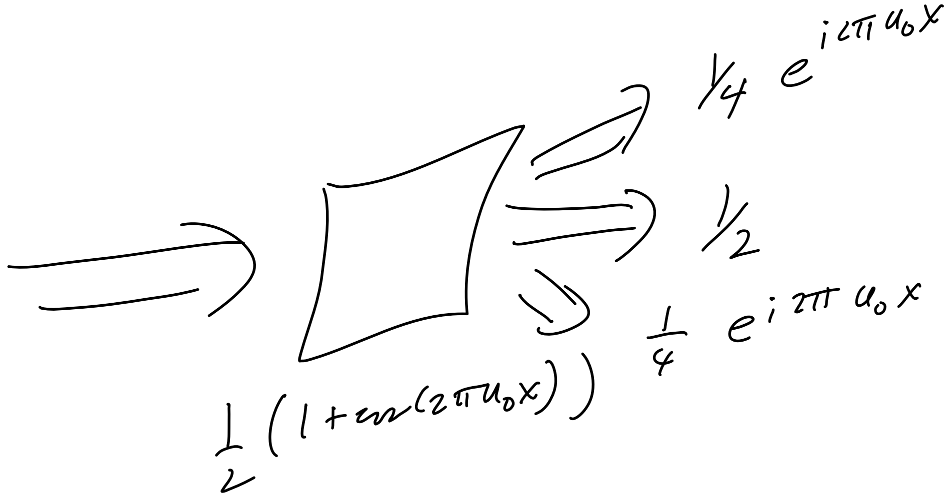
$$f_0 e^{i2\pi z}$$

$$t(x) = |A|^2 + |f_0|^2 + |A||f_0| \cos(2\pi u_0 x + \phi)$$

max modulation depth for $f_0 = A$

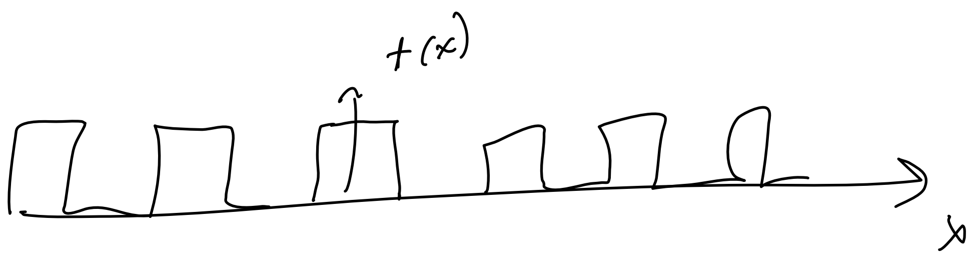
$$t = \frac{1}{2}\left(1 + \cos(2\pi u_0 x)\right)$$

Amplitude diffraction efficiency



Compare diffraction

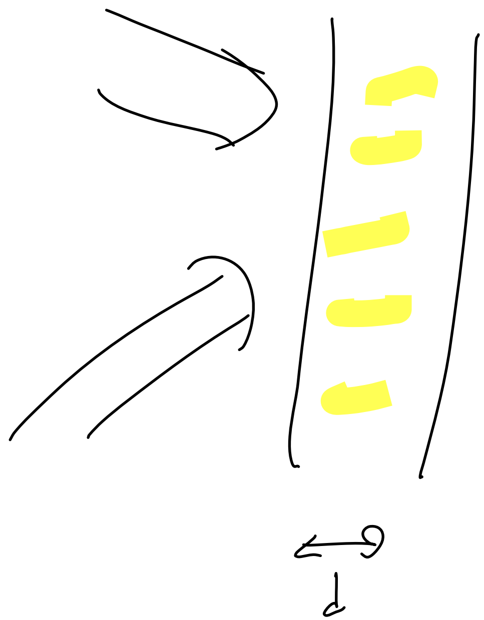
$$t(x) = \text{rect}\left[\cos(\pi u_0 x)\right] = \sum_{n=-\infty}^{\infty} \frac{\sin \pi n/2}{\pi n} e^{i2\pi n u_0 x}$$



Phase hologram

recording modulates index of refraction

n



$$\Delta n = |r|^2 + |f|^2 + rf^* + r^* f$$

$$\Delta\phi = 2\pi \frac{d}{\lambda} \Delta n$$

$$t(x,y) = e^{i\Delta\phi(x,y)}$$

Example

$$\Delta n = \Delta n_0 \sin(2\pi u_0 x)$$

$$t(x,y) = e^{i 2\pi \Delta n_0 \frac{d}{\lambda} \sin(2\pi u_0 x)}$$

$$= \sum_{q=-\infty}^{\infty} J_q\left(\frac{2\pi \Delta n_0 d}{\lambda}\right) e^{i 2\pi u_0 q x}$$

Jacobi - Anger expansion

BesselJ[0, 1] = .76

1 1 = .44

2 1 = 0.11

3 1 = 0.019